Fall 24

MATH 5A - TEST 3 v3
(3.1-3.5, 3.7)

100 points **NAME:**_______________

YOU MUST SHOW YOUR WORK.
Correct notation must be used. Phones must be OFF and put away. No graphing calculators allowed.

CIRCLE T FOR TRUE, F FOR FALSE. (2 points each)

T F (1) If $f'(c)=0$ then f has an local extreme value at c .

For example $x=0$ is a crit # of $y=x^3$ but does not yield a local extremum

T F (2) $f(x)=3x^5+7x+4$ is increasing for all x.

$f'(x)=15x^4+7 \geq 0$ for all x.

T F (3). If c is the only critical number for f(x), then the absolute maximum must occur at x=c.

f might not have an abs. max

T F (4) The graph of $f(x)=\dfrac{2x^3+1}{x^2+x}$ has a slant asymptote y=2x-2.

$$\begin{array}{r} 2x-2 \\ x^2+x \,\big)\, 2x^3+1 \\ -(2x^3+2x^2) \\ \hline -2x^2+1 \\ -2x^2-2x \\ \hline 2x+1 \end{array}$$

$2x-2 + \dfrac{2x+1}{x^2+1}$

T F (5) If $f'(x)$ is undefined at x=a then there is a cusp at x=a.

not necessarily, could be discont's or vertical tangent

T F (6) The graph of $f(x)=\dfrac{4x^2-5x+1}{3x^3+x}$ has a horizontal asymptote y=0

$$\lim_{x\to\infty} \frac{4x^2-5x+1}{3x^3+x} \cdot \frac{1/x^2}{1/x^2} = \lim_{x\to\infty} \frac{4-\frac{5}{x}+\frac{1}{x^2}}{3x+\frac{1}{x}} = 0$$

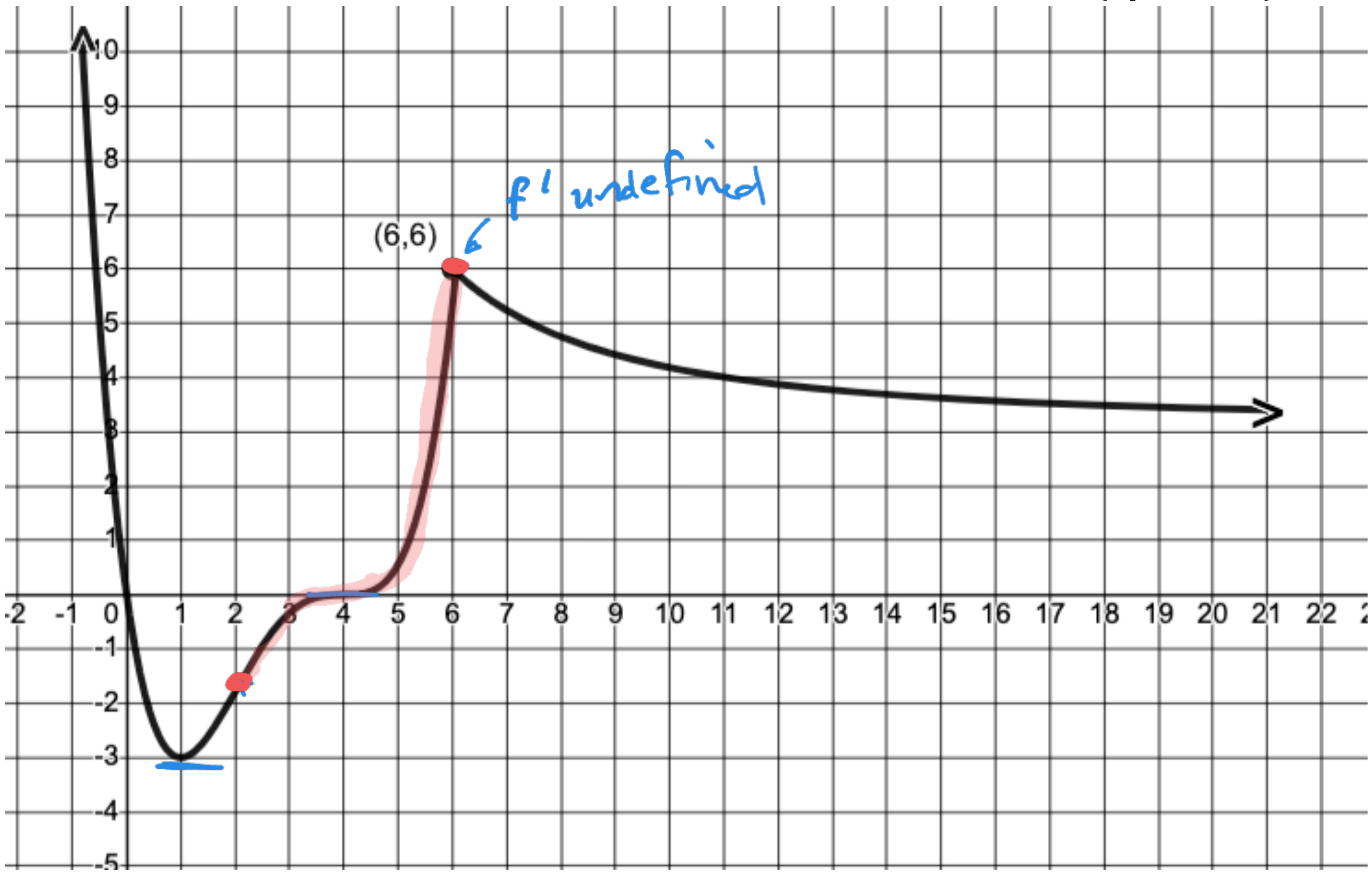
T F (7) If $f(x)$ has an absolute maximum at c then it also has a local maximum at c.

c could be at an endpoint and local extrema do not occur at endpoints

(8) In the following graph of f(x), find the following. Be sure to give specifically what is asked, no more, no less:

Assume the domain of the graph is all real numbers. (so x=-1 is not a vertical asymptote)

(2 points each)



a) What are the critical numbers for f(x)? ______ -3, 4, 6 ______

b) On what interval(s) does it appear is increasing? ______ (1,4)∪(4,6) OR (1,6) ok ______

c) $\lim_{x \to -\infty} f(x)$ ______ ∞ ______

d) On what interval(s) does it appear f is concave down? ______ (2,4) ______

e) Give the equation of any horizontal asymptote. ______ y = 3 ______

f) Give the coordinates of any local minima ______ (1, -3) ______

g). Give the coordinates of any local maxima ______ (6,6) ______

h) If we restrict the domain to [2, 8], what is the absolute maximum value, if any? ______ 6 ______

i) If we restrict the domain to [2, 8], what is the absolute minimum value, if any? ______ -2 ______

j). For what values of x, if any, does f appear not to be differentiable.? ______ x = 6 ______

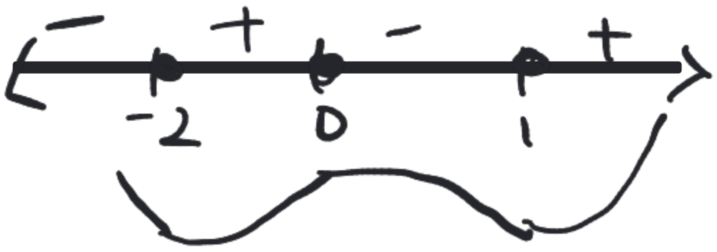
(8) Given $f(x)=\frac{1}{4}x^4+\frac{1}{3}x^3-x^2$ (18 points)

On each of the parts of this problem, you must show work to validate your claim.

(a) Find the **coordinates** of any points where local any extrema occur.

$f'(x)=x^3+x^2-2x=x(x^2+x-2)=x(x+2)(x-1)$

Crit #s : $x=0,-2,1$



Coordinates of Local Max(s) : $(0,0)$

Coordinates of Local Min(s): $(-2,\frac{-8}{3})(1,\frac{-5}{12})$

Give exact values

$f(-2)=4-\frac{8}{3}-4$

$f(1)=\frac{1}{4}+\frac{1}{3}-1=\frac{-5}{12}$

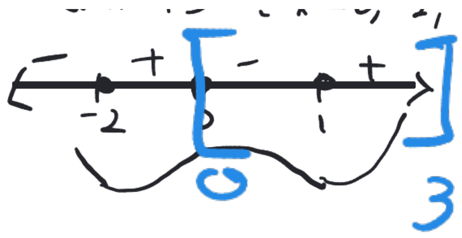
(c) Find absolute extrema (max values and min values, if any) of f(x) on the interval [0, 3].

x	0	3	1
f(x)	0	$\frac{81}{4}$	$-\frac{5}{12}$

Absolute Max: $81/4$

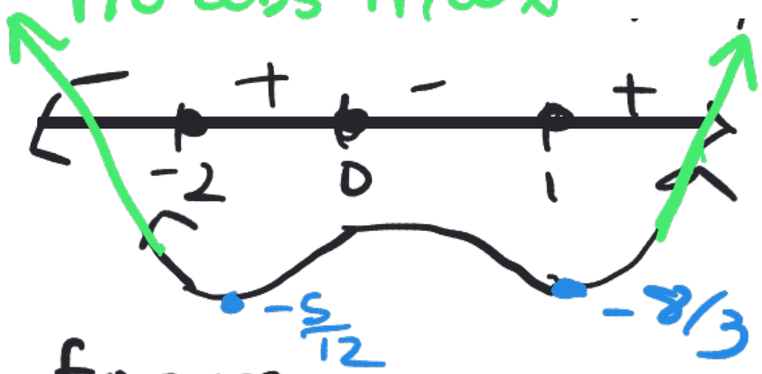
Absolute Min: $-5/12$

$f(3)=\frac{1}{4}\cdot 81+9-9=\frac{81}{4}$



(d) Find absolute extrema (max values and min values, if any) of f(x) on $(-\infty,\infty)$

no abs max



Absolute Max: none

Absolute Min: $-8/3$

for min, compare $f(-2), f(1)$

(9) (a) Does Rolle's Theorem apply to the given function? If so, find "c". If not, why not?

$f(x) = x^{2/3}$ on [-1,1] (3 points)

$f'(x) = \frac{2}{3x^{1/3}}$ not defined at $x=0$

So f not diffable at $x=0$ NO

(b) Does Rolle's Theorem apply to the given function? If so, find "c". If not, why not?

f(x) = $\sin x - \cos^2 x \;\; on \;\; \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ (5 points)

$f'(x) = \cos x + 2\cos x \sin x$

f conts + diffable on $(-\infty, \infty)$, so yes

Rolle's Thm applies

There is a $c \in \left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$ s.t $f'(c) = 0$

$\cos(c) + 2\cos(c)\sin(c) = 0$

$\cos(c)(1 + 2\sin(c)) = 0$

$\cos(c) = 0$ $\sin(c) = -1/2$

$c = \pi/2$

(c) Does the Mean Value Theorem apply to the given function? If so, find "c" as described in the theorem. If not, why not?

f(x) = $\sqrt{x-2}$, [2,6] (5 points)

$f'(x) = \frac{1}{2\sqrt{x-2}}$

f conts on [2,6] and diffabe on (2,6)

so yes. So there is a c such that

$f'(c) = \frac{f(6) - f(2)}{6-2}$

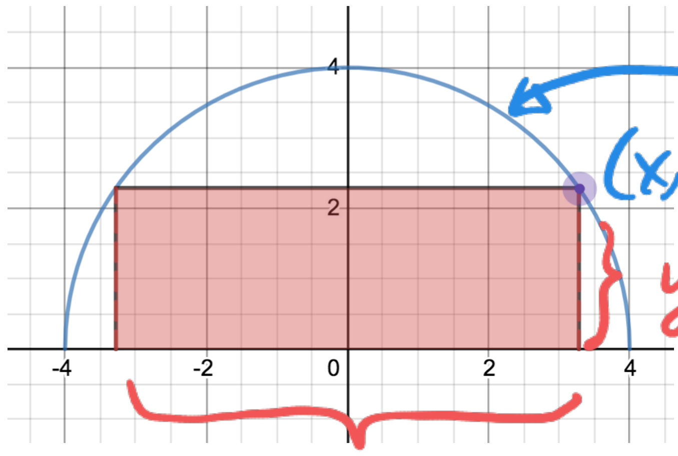
$\frac{1}{2\sqrt{c-2}} = \frac{2}{4}$

$\sqrt{c-2} = 1$

$c - 2 = 1$

$c = 3$

(10) What are the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 4? Show how you are sure you have an *absolute* max. (15 points)



(x,y) $\quad y$ $\quad 2x$

$x^2+y^2=16$

$y=\sqrt{16-x^2}$

If you don't use the closed interval method, you have to validate your abs. max somehow.

Maximize Area $= lw = 2xy$

Need in terms of one variable

so $\quad A = 2x\sqrt{16-x^2}$

$\boxed{0 \le x \le 4}$

can use closed interval method

Find crit #s

** This is done on the video for 3.7i. It is also done two different ways in 3.7 Ex5 of Book

$$A'(x) = 2\sqrt{16-x^2} + 2x \cdot \tfrac{1}{2}(16-x^2)^{-1/2}(-2x)$$

$$= 2(16-x^2)^{1/2} - 2x^2(16-x^2)^{-1/2}$$

$$= 2(16-x^2)^{-1/2}\left(16-x^2-x^2\right)$$

$$A'(x) = \frac{2(16-2x^2)}{\sqrt{16-x^2}}$$

$$A'(x)=0 \Rightarrow 16-2x^2=0$$

$$x^2=8$$

$$x=\sqrt{8}$$

	crit #	end pts	
x	$\sqrt{8}$	0	4
A(x)	16	0	0

$\Rightarrow$ Abs max $=16$ which occurs when $x=\sqrt{8}$

so length $=2x = 4\sqrt{2}$

width $= y = 2\sqrt{2}$

(11) Graph and discuss $f(x) = 2x - 3x^{2/3}$ (20 pts)

Specifically, your presentation must include discussion of any of the following which pertain to this graph: domain, intercepts, vertical and horizontal tangents, vertical and horizontal asymptotes, sign charts for f', and f'', local extrema and inflection points.

Please organize and label your work neatly so I can grade it.

graph below x axis | graph above x axis

f: $-$ 0 $-$ $\frac{27}{8}$ $+$

$\boxed{f(x)}$: $f(x) = x^{2/3}(2x^{1/3} - 3)$

domain $(-\infty, \infty)$

X-int: $x^{2/3} = 0$ $\quad 2x^{1/3} - 3 = 0$

$x = 0$ $\quad x^{1/3} = 3/2$

$x = \frac{27}{8} = 3\,3/8$

Y-int: $y = 0$

End Behavior

$\lim_{x \to \infty} x^{2/3}(2x^{1/3} - 3) = \infty$

$\lim_{x \to -\infty} x^{2/3}(2x^{1/3} - 3) = -\infty$

$\boxed{f'(x)}$

$f'(x) = 2 - 2x^{-1/3}$

$= \frac{2x^{1/3} - 2}{x^{1/3}}$

CRit #s

$f'(x) = 0 \Rightarrow 2 - 2x^{-1/3} = 0$

$x^{-1/3} = 1$

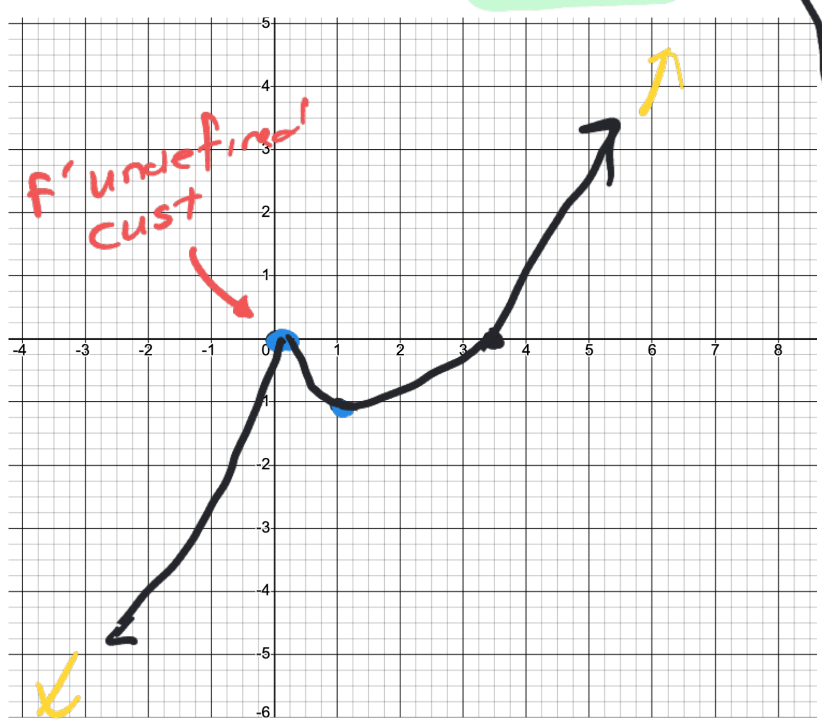
$x = 1$

$f'(x)$ undef $\quad x^{1/3} = 0$

$x = 0$

$f'(x)$: $+$ 0 $-$ 1 $+$

Horiz tan.



local max at $(0,0)$
local min at $(1,-1)$

$f''(x) = \frac{2}{3}x^{-4/3} = \frac{2}{3x^{4/3}}$

f'': $+$ cu 0 $+$ cu

No inflection